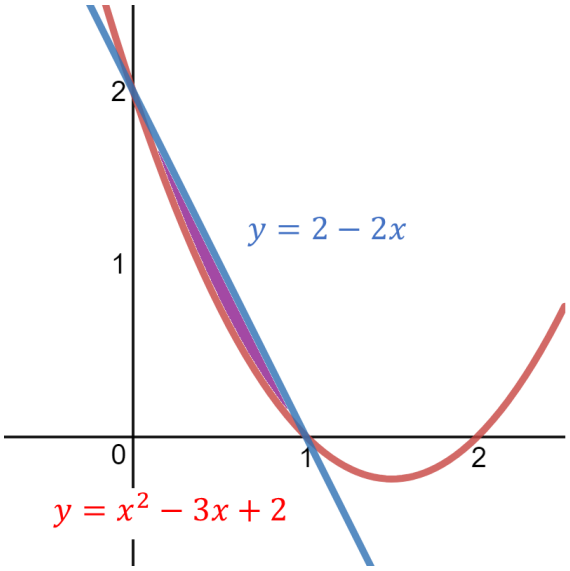
**QUIZ 10 SOLUTIONS: LESSONS 11-12**
**SEPTEMBER 24, 2018**

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

**1.** [5 pts] Find the area between

$$y = x^2 - 3x + 2 \text{ and } y = 2 - 2x.$$

**Solution**: We quickly sketch a graph of the region:




But if that is difficult to see, we can always determine our bounds algebraically. We set our functions equal to each other:

$$x^2 - 3x + 2 = 2 - 2x$$

$$x^2 - x = 0$$

$$x(x - 1) = 0$$

Hence, our bounds are $x = 0$ and $x = 1$. Further, if we check at $x = \frac{1}{2}$, we have

$$\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 2 = \frac{1}{4} - \frac{3}{2} + 2$$

$$= \frac{1}{4} - \frac{6}{4} + \frac{8}{4}$$

$$= \frac{3}{4}$$

$$2 - 2\left(\frac{1}{2}\right) = 2 - 1 = 1$$

Since $1 > \frac{3}{4}$, we see that $2 - 2x$ is the larger function. Our integral is then

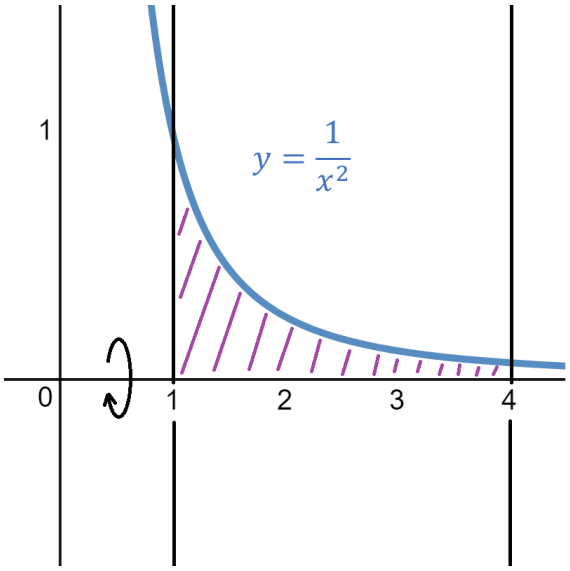
$$\begin{aligned}
\text{Area} &= \int_0^1 \left[2 - 2x - (x^2 - 3x + 2)\right]\, dx \\
&= \int_0^1 \left[2 - 2x - x^2 + 3x - 2\right]\, dx \\
&= \int_0^1 \left[x - x^2\right]\, dx \\
&= \frac{1}{2}x^2 - \frac{1}{3}x^3\Big|_0^1 \\
&= \frac{1}{2}(1)^2 - \frac{1}{3}(1)^3 \\
&= \frac{1}{2} - \frac{1}{3} \\
&= \frac{3}{6} - \frac{2}{6} = \boxed{\frac{1}{6}}
\end{aligned}$$

**2.** [5 pts] Find the volume of the solid generated by revolving the region bounded by

$$y = \frac{1}{x^2}, \quad y = 0, \quad x = 1, \quad x = 4$$

about the $x$-axis.

**Solution**: Again, we sketch a picture:




This is a disk method problem. Our integral is

$$\begin{aligned}
\text{Vol} &= \pi \int_1^4 \left[ \left( \frac{1}{x^2} \right)^2 \right] dx \\
&= \pi \int_1^4 \left[ \frac{1}{x^4} \right] dx \\
&= \pi \left[ -\frac{1}{3x^3} \right]_1^4 \\
&= \pi \left[ -\frac{1}{3(4)^3} - \left( -\frac{1}{3(1)^3} \right) \right] \\
&= \pi \left[ -\frac{1}{3 \cdot 64} + \frac{1}{3} \right] \\
&= \pi \left[ -\frac{1}{3 \cdot 64} + \frac{64}{3 \cdot 64} \right] \\
&= \pi \left[ \frac{63}{3 \cdot 64} \right] \\
&= \boxed{\frac{63\pi}{192}}
\end{aligned}$$